

MATHS
CALCULATIONS
BOOKLET

Mathematics at Twydall Primary

We have produced this booklet to allow parents to better understand how maths calculations are delivered at Twydall Primary School.

Our aim is to develop mathematicians who are engaged, inspired and confident. To achieve this, a strong and assured grasp of the four number operations (addition, subtraction, multiplication and division) is important for both written methods as well as mental calculations.

This booklet contains the stages in teaching for each mathematical operation. In any given class or year group children will be working at different stages and will only move on when it is appropriate for them.

Essential Ideas

Children should

- Estimate calculations
- Consider whether to calculate mentally or using a written method
- Check answers: Is it reasonable? Can I do the inverse?
- Use equipment when it is helpful: counters, bead string, place value cards
- Be fluent in knowing times tables facts : children should aim to know:
 - ✓ $\times 2$, $\times 5$ and $\times 10$ and related division facts by the end of year 2;
 - ✓ $\times 3$, $\times 4$ and $\times 6$ and related division facts by the end of year 3;
 - ✓ all tables including $\times 7$, $\times 8$, $\times 9$ by the end of year 4

Addition

Stage 1

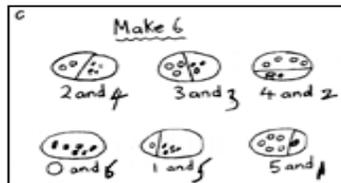
Practical activities and discussions e.g. Using two objects, how can I make a given total?

Finding one more than a number from 1 to 10

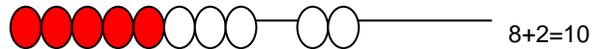
Using vocabulary associated with addition questions should be real life and related to children's experiences

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.

They develop ways of recording calculations using pictures, etc.



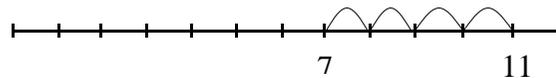
Bead strings or bead bars can be used to illustrate addition



They use numberlines and practical resources to support calculation and teachers demonstrate the use of the numberline.

Number lines (blank) with teacher support

$$7 + 4 = 11$$



Children go up in 1s

+ = signs and missing numbers

$$\begin{array}{ll} 3 + 4 = \square & \square = 3 + 4 \\ 3 + \square = 7 & 7 = \square + 4 \\ \square + 4 = 7 & 7 = 3 + \square \\ \square + \nabla = 7 & 7 = \square + \nabla \end{array}$$

Show that $3 + 4$ is the same as 7 using 2 separate bead strings.

Stage 2

+ = signs and missing numbers

Extend to

$$14 + 5 = 10 + \square$$

Opportunities to explore place value

and adding three numbers

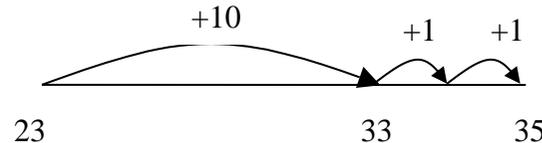
$$32 + \square + \square = 100 \quad 35 = 1 + \square + 5$$

Partition into tens and ones and recombine

$$\begin{aligned} 12 + 23 &= 10 + 2 + 20 + 3 \\ &= 30 + 5 \\ &= 35 \end{aligned}$$

refine to partitioning the second number only:

$$\begin{aligned} 23 + 12 &= 23 + 10 + 1 + 1 \\ &= 33 + 1 + 1 \\ &= 35 \end{aligned}$$



This can also be represented using more than 1 bead string (23 on one and 12 on the other) so that the colours can be used to identify tens and ones.

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

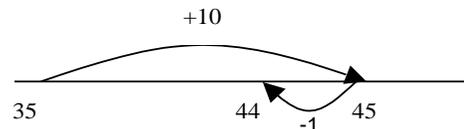


Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones. More confident learners can begin to use an empty.

Mental Method:

Add 9 or 11 by adding 10 and adjusting by 1

$$35 + 9 = 44$$



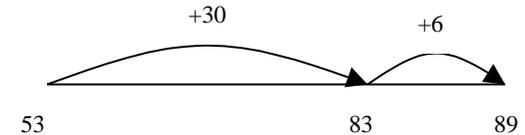
Stage 3

+ = signs and missing numbers

Partition into tens and ones and recombine

Partition **both** numbers and recombine. Refine to partitioning the second number only e.g.

$$\begin{aligned} 36 + 53 &= 53 + 30 + 6 \\ &= 83 + 6 \\ &= 89 \end{aligned}$$



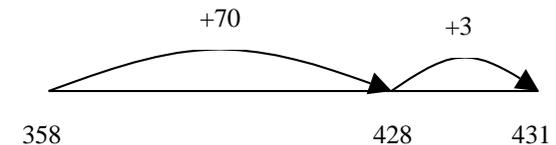
Add a near multiple of 10 to a two-digit number

Partition into hundreds, tens and ones and recombine

With larger numbers children can use their individual reasoning when partitioning

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 358 + 73 &= 358 + 70 + 3 \\ &= 428 + 3 \\ &= 431 \end{aligned}$$



Addition

Stage 4

Children must be confident within their understanding of the value of each digit.

They need to add the ones first. Therefore they will know how many tens they might have, from adding the ones. They can then add this to the total from the tens column.

Record steps in addition using partitioning:

Partitioned numbers are then written under one another:

$$\begin{array}{r}
 47 \\
 + 76 \\
 \hline
 \end{array}
 =
 \begin{array}{r}
 40 + 7 \\
 70 + 6 \\
 \hline
 110 + 13 = 123
 \end{array}$$

Add the ones first as this will help the children with later calculations as they move through the progression.

Using more formal columns (pairs of 2 digit numbers)

$$47 + 76 = 123$$

Write the larger number written on top

Add the ones first

$$\begin{array}{r}
 76 \\
 + 47 \\
 \hline
 13 \\
 110 \\
 \hline
 123
 \end{array}$$

NB vocab: use 70 + 40, not 7 + 4

$$\begin{array}{r}
 358 \\
 + 73 \\
 \hline
 11 \\
 120 \\
 \hline
 300 \\
 \hline
 431
 \end{array}$$

Stage 5

Pencil and paper procedures

Leading to formal method, showing numbers carried underneath

$$\begin{array}{r}
 358 \\
 + 73 \\
 \hline
 431 \\
 11
 \end{array}$$

Extend to numbers with **at least four digits**

$$3587 + 675 = 4262$$

$$\begin{array}{r}
 3587 \\
 + 675 \\
 \hline
 4262 \\
 111
 \end{array}$$

Extend to decimals (same number of decimals places) and adding several numbers (with different numbers of digits).
Model negative numbers using a number line.

Stage 6

Pencil and paper procedures

Extend to numbers with any number of digits and decimals with 1 and 2 decimal places.

$$124.9 + 117.25 = 242.15$$

$$\begin{array}{r}
 124.90 \\
 + 117.25 \\
 \hline
 242.15 \\
 11
 \end{array}$$

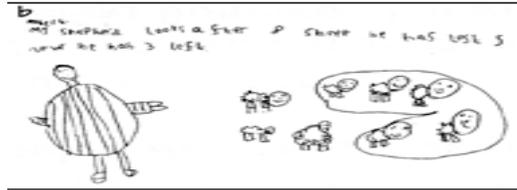
add in a zero to keep the place value digits of similar value are in line.

Subtraction

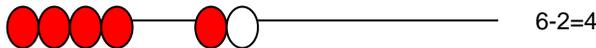
Stage 1

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.

They develop ways of recording calculations using pictures etc.



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.



They use numberlines and practical resources to support calculation. Teachers demonstrate the use of the numberline to develop visual links with a bead string.

- = signs and missing numbers

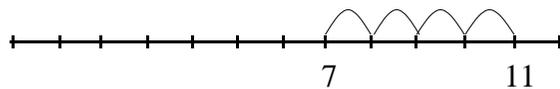
$7 - 3 = \square$	$\square = 7 - 3$
$7 - \square = 4$	$4 = \square - 3$
$\square - 3 = 4$	$4 = 7 - \square$
$\square - \nabla = 4$	$4 = \square - \nabla$

Visual / practical activities

Number lines

The difference between 7 and 11
(Counting on)

To reinforce concept. Practical strategies essential to see 'difference'.



Recording by - drawing jumps on prepared lines
- constructing own lines, if appropriate

(Teachers model jottings appropriate for larger numbers)

Stage 2

Bead strings can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.



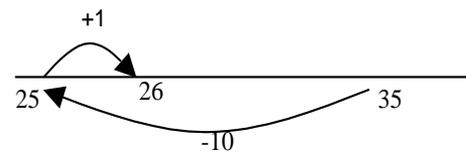
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

- = signs and missing numbers

Continue using a range of equations with appropriate numbers.

Extend to $14 + 5 = 20 - \square$

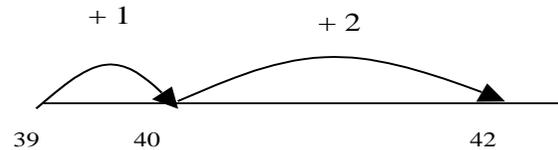
Find a small difference by counting on $42 - 39 = 3$



Mental Method

Subtract 9 or 11. Begin to add/subtract 19 or 21

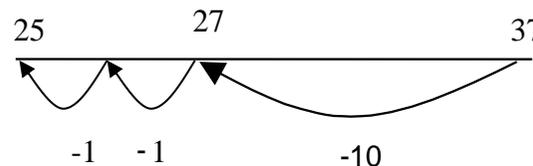
eg. $35 - 9 = 26$



Use known number facts and place value to subtract

(partition second number only)

$$\begin{aligned} 37 - 12 &= 37 - 10 - 2 \\ &= 27 - 2 \\ &= 25 \end{aligned}$$



Stage 3

Find a small difference by counting on

Use of larger numbers (hundreds) e.g. $102 - 97 = 5$

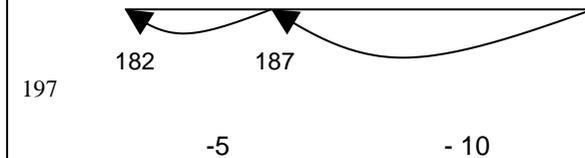
Use known number facts and place value to subtract

e.g. 3 digit number - 2 digit number

Estimate first....

$$200 - 20 = 180$$

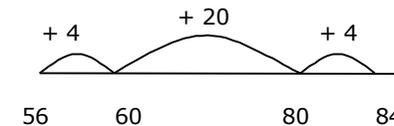
$$197 - 15 = 182$$



Pencil and paper procedures

Complementary addition and mental instant recall facts eg. number bonds, adding tens

$$84 - 56 = 28$$



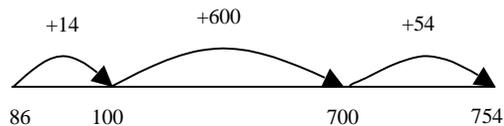
Subtraction

Stage 4

Pencil and paper procedures

Complementary addition

$$754 - 86 = 668$$



$$\begin{array}{r} 98 \\ - 24 \\ \hline 4 \quad (8-4) \\ 70 \quad (90-20) \\ \hline 74 \end{array}$$

Use decomposition with confident children when appropriate

$$\begin{array}{r} 92 \\ - 38 \\ \hline 54 \end{array} \quad (\text{explain what happens...see below})$$

$$\begin{array}{r} 90 + 2 \quad \rightarrow \quad 80 + 12 \\ 30 + 8 \quad - \quad 30 + 8 \\ \hline 50 + 4 \end{array}$$

Stage 5

Find a difference by counting on

e.g. $8006 - 2993 = 5013$

This can be modelled on an empty number line

Pencil and paper procedures

$$\begin{array}{r} 8 \quad 1 \\ \cancel{9}2 \\ - 38 \\ \hline 54 \end{array}$$

Develop the stages of decomposition introducing 'zero'

$$\begin{array}{r} 2 \quad 4 \quad 1 \\ \cancel{3}52 \\ - 178 \\ \hline 174 \end{array} \quad \begin{array}{r} 4 \quad 9 \quad 9 \quad 1 \\ \cancel{5}000 \\ - 457 \\ \hline 4543 \end{array}$$

Stage 6

- = signs and missing numbers

Pencil and paper procedures

Develop the use decomposition

extend to up to 2 decimal places

$$48.42 - 37.61 =$$

$$\begin{array}{r} 48.42 \\ - 37.61 \\ \hline 10.81 \end{array}$$

extend to up to 3 decimal places if appropriate

$$302.63 - 178.124 =$$

$$\begin{array}{r} 2 \quad 9 \quad 1 \\ \cancel{3}02.6 \quad \cancel{10} \\ - 178.124 \\ \hline 124.506 \end{array}$$

Multiplication

Stage 1

Pictures and symbols

There are 3 sweets in one bag.

How many sweets are there in 5 bags?



Use of bead strings to model groups of.

(Recording on a number line modelled by the teacher when solving problems)

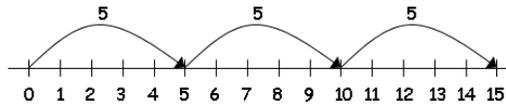
Modelling to begin to show the link between multiplication and repeated addition. Emphasise use of vocabulary to develop this.

Repeated addition

3 times 5 is $5 + 5 + 5 = 15$ or **3 lots of 5** then introduce multiplication sign 5×3

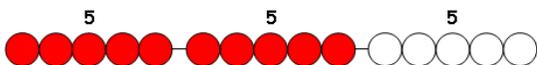
Repeated addition can be modelled on a number line:

$$5 \times 3 = 5 + 5 + 5$$



Children can use a bead string independently:

$$5 \times 3 = 5 + 5 + 5$$



Stage 2

Use of a bead string to investigate

x = signs and missing numbers

$$\begin{array}{ll} 7 \times 2 = \square & \square = 2 \times 7 \\ 7 \times \square = 14 & 14 = \square \times 7 \\ \square \times 2 = 14 & 14 = 2 \times \square \\ \square \times \nabla = 14 & 14 = \square \times \nabla \end{array}$$

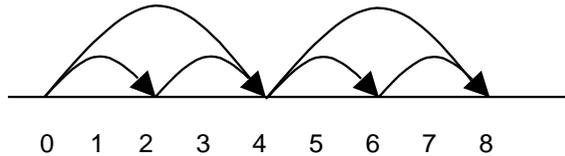
Arrays and repeated addition

$$\begin{array}{cccc} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{array} \quad 4 \times 2 \text{ or } 4 + 4$$

$$2 \times 4$$

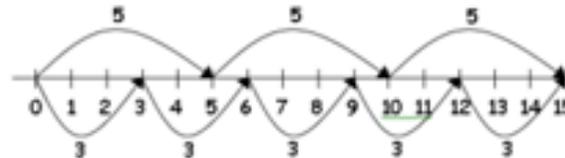
or repeated addition

$$2 + 2 + 2 + 2$$



Commutativity

Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



Stage 3

Doubling multiples of 5 up to 50

$$15 \times 2 = 30$$

Partition

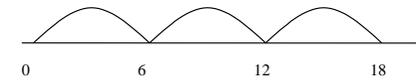
$$(10 \times 2) + (5 \times 2) \\ 20 + 10 = 30$$

x = signs and missing numbers

Continue using a wider range of equations.

Number lines

$$6 \times 3$$



$$35 \times 2 = 70$$

Partition

$$\begin{array}{r|l|l} x & 30 & 5 \\ \hline 2 & 60 & 10 \end{array} = 70$$

Multiplication

Stage 4

Pencil and paper procedures

Grid method
 TU x U
 $36 \times 4 =$

	Tens	Ones
X	30	6
4	120	4

$120 + 24 = 144$ (add the partial products)
 $36 \times 4 = 144$

HTU x U
 $123 \times 3 = 369$

	<small>H</small>	<small>T</small>	<small>O</small>
X	100	20	3
3	300	60	9

Note Change in orientation for larger numbers

Stage 5

Pencil and paper procedures

Grid method
 72×38

Estimate and check

is approximately $70 \times 40 = 2800$

x	70	2	=	2160
30	2100	60	=	<u>576</u> +
8	560	16		<u>2736</u>
				1

Moving on to formal method when appropriate.
 'Carried' numbers to sit on top line of answer box

$1125 \times 7 = 7875$

	<small>Th</small>	<small>H</small>	<small>T</small>	<small>O</small>
X	1000	100	20	5
7	7000	700	140	35

Formal method for long multiplication

	<small>1</small>		
	7	2	
X	3	8	
	5	7	6
+	2	1	6
	2	7	3
			6
	<small>1</small>		

Note:
 When multiplying by 8
 $2 \times 8 = 16$ $70 \times 8 = 560$
 when multiplying by 30
 $2 \times 30 = 60$ $70 \times 30 = 2100$

From 2016 end of KS2 assessments, only evidence of long multiplication will gain working marks (assuming errors in the final answer)

Stage 6

x = signs and missing numbers

Pencil and paper procedures
 Grid method
 Estimate and check

372×24 is approximately $400 \times 20 = 8000$

	300	70	2
x	6000	1400	40
20	1200	280	8
4			

<small>th</small>	<small>h</small>	<small>t</small>	<small>u</small>
	<small>1</small>		
	6	0	0
	1	4	0
	1	2	0
		2	8
			4
			8
			8

Grid method for decimals

7.2×3.8

	7	0.2
x	21	0.6
3	5.6	0.16
0.8		

8 9 2 8

= 21.60
 = 5.76 +
27.36
1

Only for children who already know this method (and are accurate with it).

Division

Stage 1

Pictures / marks

12 children get into teams of 4 to play a game. How many teams are there?



Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.



$$12 \div 3 =$$

Stage 2

Understand division as sharing and grouping

Sharing – 6 sweets are shared between 2 people. How many do they have each?



$6 \div 2$ can be modelled as:

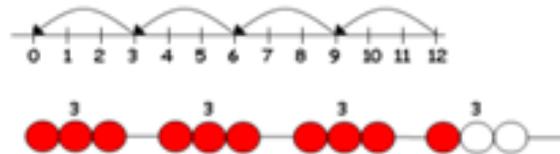
Grouping or repeated subtraction

There are 6 sweets, how many people can have 2 sweets each?



Repeated subtraction using a number line or bead string

$$12 \div 3 = 4$$



This will help interpret the statement 'how many 3s make 12'

Using symbols

Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \div 2 = 4 \quad 20 \div \triangle = 4 \quad \square \div \triangle = 4$$

Stage 3

\div = signs and missing numbers

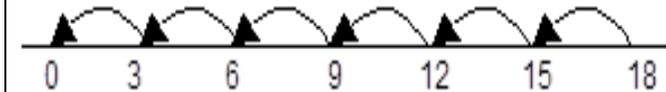
Continue using a range of equations as in Level 2 but with appropriate numbers.

Understand division as sharing and grouping

$18 \div 3$ can be modelled as:

Sharing – 18 shared between 3 (see Level 2 diagram)

Grouping - How many 3's make 18?



Remainders

Sharing - 16 shared between 3, how many left over?

Grouping – How many 3's make 16, how many left over?

e.g.

$$16 \div 3 =$$



$$16 \div 3 = 5 \text{ r}1$$

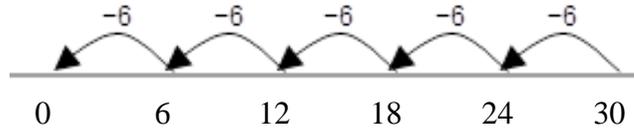
Division

Stage 4

\div = signs and missing numbers

Sharing and grouping

$30 \div 6$ can be modelled as:
grouping – groups of 6 taken away and the number of groups counted
e.g. sharing among 6, the number given to each person



Grouping

$$41 \div 4 = 10 \text{ r}1$$



OR $41 = (10 \times 4) + 1$

Stage 5

\div = signs and missing numbers

Remainders

Quotients expressed as fractions or decimal fractions
 $61 \div 4 = 15 \frac{1}{4}$ or 15.25

Pencil and paper procedures

BUS STOP METHOD

$$\begin{array}{r} 215.25 \\ 4 \overline{) 8621.1020} \end{array}$$

Break into steps instead of chunking

Stage 6

\div = signs and missing numbers

Remainders

Quotients expressed as fractions or decimal fractions using larger numbers

$$676 \div 8 = 84.5$$

Pencil and paper procedures

$977 \div 36$ is approximately $1000 \div 40 = 25$

Using chunking for division of larger number and dividing by 2-digit numbers.

BUS STOP METHOD

$$\begin{array}{r} 027.1388 \\ 36 \overline{) 977.0000} \\ \underline{0} \\ 97 \\ - 72 \\ \underline{257} \\ - 252 \\ \underline{45.10} \\ 3.6 \\ \underline{1.34} \quad 10 \\ 1.08 \\ \underline{0.23} \quad 112 \quad 10 \\ \quad 288 \\ \quad \underline{320} \end{array}$$